

Computational Solid Mechanics / WMME028-05

Written exam, 5 April 2024, 8:30 - 10:30

Write your name and student number on each page

Exercise 1

Consider the Saint Venant model

$$\mathbf{S} = \lambda \text{tr}(\mathbf{E})\mathbf{I} + 2\mu\mathbf{E} , \quad (1)$$

where $\mathbf{S}$ is the second Piola-Kirchhoff stress tensor, $\mathbf{I}$ is the second order identity tensor, $\mathbf{E}$ is the Green-Lagrange strain, λ and μ are the Lamé elastic constants. The material undergoes uniaxial stretch $\mathbf{F} = \mathbf{I} + (\bar{\lambda} - 1)\mathbf{e}_1 \otimes \mathbf{e}_1$, where $\bar{\lambda}$ is the stretch.

1. Compute the first Piola-Kirchhoff stress tensor for this deformation (1.5 point) [15 minutes];
2. Consider a 1D bar under tension. The 1D constitutive equation is defined by the component P_{11} of the 3D first Piola-Kirchhoff stress tensor. Compute the analytical tangent $\frac{\partial P_{11}}{\partial \lambda}$ (0.5 point) [5 minutes];
3. Pick two values of the stretch $\bar{\lambda}$ of your choice, and evaluate the analytical tangent. Will it change? Explain (0.5 point) [5 minutes].
4. Consider a FEM discretization of this 1D bar, consisting of quadratic isoparametric elements with length $\ell = 2$. Compute one component of the element stiffness matrix (2 points) [30 minutes];

Exercise 2

Consider the FEM mesh in Figure 1, including two 2D Quad4 bilinear isoparametric elements.

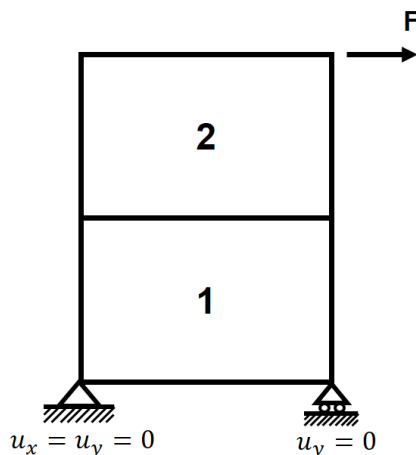


Figure 1: **2D FEM mesh.**

1. Write down the connectivity matrix (0.5 point) [5 minutes];
2. After assembly, which slots of the global stiffness matrix will be non-zero? (1.5 point) [20 minutes];
3. Partition the global stiffness matrix: which slots will belong to which submatrix? (1 point) [15 minutes].

[Hint: To answer the questions, represent the matrix as a checkerboard].

Exercise 3

Consider the function

$$f(x) = x^2 - 3x - 4 \quad , \quad x \in \left(-\infty, \frac{3}{2}\right) . \quad (2)$$

Start from the initial guess $x_0 = -3$. Use the Newton-Raphson method to find the zero of the function within an absolute tolerance of the unknown equal to 0.05 (2.5 points) [25 minutes].