

(1) EXAM 4 solc.

A)  $A \frac{dh}{dt} = v_2 A_2 - v_1 A_1$

$v_1 = (2gh)^{1/2}$  Apply Bernoulli!

Steady-state:  $v_2 A_2 = (2gh)^{1/2} \cdot A_1$

$h = \frac{v_2^2}{2g} \cdot \left(\frac{A_2}{A_1}\right)^2$

$= \frac{25}{20} \cdot 4 = 5 \text{ m}$  AND  $v_1 = 10 \text{ m/s}$

B)  $V \frac{dc}{dt} = v_2 A_2 c_{in} - v_1 A_1 c$

$V = A \cdot h = 100 A_1 \cdot 5 = 500 A_1$

~~$500 A_1 \frac{dc}{dt} = 5 \times 2 \times A_1 \cdot 10 - 10 \cdot A_1 \cdot c$~~

$50 \frac{dc}{dt} = 10 - c$

$\int_0^c \frac{dc}{10-c} = \frac{1}{50} \int_0^t dt$

$\ln \frac{10}{10-c} = \frac{t}{50}$

$c(t) = 10 - 10 \exp(-\frac{t}{50})$

$$(2) A) V_{CR} = f(\rho, g, \sigma, d)$$

$$V_{CR} = K \rho^a \cdot g^b \cdot \sigma^c \cdot d^d$$

$$m^3 = \left(\frac{kg}{m^3}\right)^a \cdot \left(\frac{m}{s^2}\right)^b \cdot \left(\frac{kg}{s^2}\right)^c \cdot m^d$$

$$\begin{array}{l} m: \quad 3 = -3a + b + d \\ kg: \quad 0 = a + c \\ s^2: \quad 0 = -2b - 2c \end{array} \quad \begin{array}{l} d = 3 - 2c \\ a = -c \\ b = -c \end{array}$$

$$V_{CR} = K \cdot \rho^{-c} \cdot g^{-c} \cdot \sigma^c \cdot d^{3-2c}$$

$$\frac{V_{CR}}{d^3} = K \left( \frac{\rho \cdot g \cdot d^2}{\sigma} \right)^{-c}$$

$\pi$ -Buckingham:  $5 - 3 = 2$  dimensionless groups

$$B) V_{CR} \sim d \rightarrow c = 1$$

$$V_{CR} = K \frac{\sigma d}{\rho \cdot g}$$

Substitute values:  $K = \pi = 3.14$

$$C) \frac{\sigma}{\rho} \text{ is constant so } \frac{V_{CR,1}}{V_{CR,2}} = \frac{\sigma_1 \rho_2}{\sigma_2 \rho_1}$$

3) A)  $V \cdot \rho \cdot c_p \frac{dT}{dt} = -U \cdot A (T - T_R)$

$$\int_{32}^{26} \frac{dT}{T - T_R} = - \frac{U \cdot A}{V \cdot \rho \cdot c_p} \int_0^{120} dt$$

$$\frac{A}{V} = \frac{\pi D \cdot h}{\frac{\pi}{4} D^2 \cdot h} = \frac{4}{D}$$

$$\ln \frac{26 - 20}{32 - 20} = - \frac{U \cdot 4}{2 \times 2 \cdot 10^3 \times 0.10^3} \times 120$$

$$\ln 2 = U \times 15 \cdot 10^{-6}$$

$$U = \frac{0.69}{15} \times 10^6 \approx 4.5 \cdot 10^4$$

B)  $A/V = 4/D$  independent of  $h$ !

SAME TIME, 2 HRS

④

$$A) v_1 A_1 = v_2 A_2 \rightarrow v_2 = 6 \text{ m/s}$$

Bernoulli to calculate  $p_2$

$$\frac{p_1}{\rho} + \frac{1}{2} v_1^2 = \frac{p_2}{\rho} + \frac{1}{2} v_2^2$$

$$\frac{20}{0.8} + 4.5 = \frac{p_2}{0.8 \cdot 10^3} + 18$$

$$p_2 = \frac{11.5}{0.8} \times 0.8 \cdot 10^3 = 11.5 \cdot 10^3 \text{ Pa}$$

$$B) \frac{d(Mv_x)}{dt} = A_1 \rho v_{x1}^2 - A_2 \rho v_{x2}^2 + \sum F_x$$

$$\sum F_x = p_1 A_1 + p_2 A_2 + F_{x, \text{wall} \rightarrow \text{fluid}}$$

↑ ON the system

↑ Reaction force in positive x-direction

Steady-state

$$0 = A_1 \rho v_1^2 + A_2 \rho v_2^2 + p_1 A_1 + p_2 A_2 + F_{x, \text{wall} \rightarrow \text{fluid}}$$

↑ outflow in negative x-direction

Filling in the numbers:  $F_{x, \text{wall} \rightarrow \text{fluid}} = -50 \cdot 10^3 \text{ N}$

$F_{x, \text{wall}} = 50 \cdot 10^3 \text{ N}$  in positive x-direction

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$$V \frac{dc}{dt} = \phi_v c|_{in} - \phi_v c|_{out} + \frac{2}{x} V$$

Asked concentration profile  $\rightarrow$

micro balance!

Steady-state:

$$0 = v \cdot \frac{\pi}{4} D^2 c|_x - v \cdot \frac{\pi}{4} D^2 c|_{x+dx} - \frac{2}{x} \frac{\pi}{4} D^2 dx$$

$$r_x = k \cdot c \quad \text{first order!}$$

$$v \left( \frac{c}{x} - \frac{c}{x+dx} \right) = k \cdot c \cdot dx$$

$$\frac{dc}{dx} = - \frac{k}{v} \cdot c$$

$$\ln c = - \frac{k}{v} x + C_1$$

$$x=0 \quad c=c_0 \rightarrow C_1 = \ln c_0$$

$$c(x) = c_0 \exp\left(-\frac{k}{v} \cdot x\right)$$

