

Homework I: DAQ system, Auto/Cross Correlation, and DFT

Q1 [2 points] A chemical plant requires real-time monitoring of the temperature inside a reactor vessel. The temperature sensor outputs a voltage signal ranging from 0 to 10V, corresponding to a temperature range of 0 to 200°C. The system uses a 16-bit Successive Approximation Register (SAR) ADC with a reference voltage $V_{ref} = 10\text{ V}$. The system specifications demand a temperature measurement error not exceeding $\pm 0.5^\circ\text{C}$ and a sampling frequency suitable for real-time monitoring.

Note: Quantization Error: The maximum possible error introduced by the ADC due to finite resolution, equal to $\pm \frac{1}{2}$ of the least significant bit (LSB).

Questions:

- 1) [0.5 point] Calculate the **temperature resolution** ($^\circ\text{C}$) of the ADC
- 2) [0.5 point] Determine the **maximum quantization error** ($^\circ\text{C}$) and verify if the ADC meets the $\pm 0.5^\circ\text{C}$ error requirement
- 3) [0.5 point] The ADC outputs the binary code 1011001100110101. Calculate the corresponding temperature value ($^\circ\text{C}$) to one decimal place
- 4) [0.5 point] If the sampling frequency = 100Hz, determine the **cutoff frequency** of the anti-aliasing filter and justify your choice

Q2 [2 points] A bridge health monitoring system uses vibration sensors to detect hidden cracks. The sensor output is a noisy discrete-time signal $x[n]$ with 6 samples:

$$x[n] = [1, -2, 3, 0, -1, 4] \text{ for } n = 0, 1, 2, 3, 4, 5$$

The system uses autocorrelation to identify periodic vibrations caused by cracks, even when masked by noise.

Questions:

- 1) [1 point] Compute the biased autocorrelation $R_{xx}[k]$ for $k = -2, -1, 0, 1, 2$
- 2) [1 point] If Gaussian noise $\sigma = 2$ is added to $x[n]$ derive the theoretical expression for $R_{yy}[k]$ where $y[n] = x[n] + \text{Gaussian noise}$

Q3) [2 points] A **radar system** transmits a coded pulse $s[n] = [1, -1, 2]$ and receives a delayed/noisy signal $y[n] = [0, 0, 1, -1, 2, 0]$. Cross-correlation is used to estimate target distance and suppress noise.

Questions:

- 1) [1 point] Compute the unbiased cross-correlation $R_{sy}[k]$ for $k = -2, -1, 0, 1, 2$
- 2) [1 point] Identify the peak location and calculate the target distance if each sample corresponds to 150 m.

Q4) [2 points] A digital signal processor samples a signal $x(t) = \cos(2\pi \cdot 25t)$ at $f_s = 100$ Hz. The acquired discrete-time signal is truncated to 4 samples: $x[n] = [1, 0, -1, 0]$ for $n = 0, 1, 2, 3$.

Questions:

- 1) [1 point] Compute the 4-point DFT $X[k]$ for $k = 0, 1, 2, 3$
- 2) [0.5 point] Plot the two-sided magnitude spectrum $|X[k]|$, labeling frequency bins in Hz
- 3) [0.5 point] Convert the two-sided spectrum to a one-sided spectrum

Q5) [2 points] A music synthesizer generates a signal containing a 440 Hz tone and its 880 Hz harmonic. The signal is sampled at $f_s = 8$ kHz. A 10 ms duration of the signal is analyzed using DFT.

- 1) [0.5 point] What is the Nyquist frequency? Can the system capture both 440 Hz and 880 Hz without aliasing?
- 2) [0.5 point] Calculate the number of samples used in the DFT and calculate the frequency resolution of the DFT for the 10 ms signal.
- 3) [0.5 point] Identify which frequency bin corresponds to 440 Hz
- 4) [0.5 point] Is there spectral leakage when performing DFT? If yes, which measures can be taken to mitigate it?