

Solid Mechanics (for IEM)

WBIE055-05

Written Exam (2 hours)

Exercise 1 Consider a deformation with gradient $\mathbf{F} = \mathbf{e}_1 \otimes \mathbf{e}_1 + \frac{1}{\sqrt{\lambda}} \mathbf{e}_2 \otimes \mathbf{e}_2 + \sqrt{\lambda} \mathbf{e}_3 \otimes \mathbf{e}_3$, with $\mathbf{e}_i$ ($i = 1, 2, 3$) defining an orthonormal vector basis and λ a stretch parameter.

1. Determine the relative volume change for this deformation (0.5 point); [5 minutes]

2. Pick two distinct unit normals and compute their respective oriented area change (1 point); [15 minutes]

3. What are the eigenvalues and eigenvectors associated with this deformation gradient? Why? (0.5 point). [5 minutes]

4. Determine the Green-Lagrange strain tensor associated with this deformation (1 point). [10 minutes]

Exercise 2 Consider the deformation

$$\mathbf{y} = (\gamma x_2 + x_1)\mathbf{e}_1 + x_2\mathbf{e}_2 + (\lambda x_3 - \gamma x_1)\mathbf{e}_3.$$

of a linear elastic material.

1. Compute the strain and small rotation tensors (1 point); [10 minutes]
2. Compute the stress tensor (1 point); [15 minutes]
3. Compute the traction acting on a plane with normal $\mathbf{n} = \frac{1}{\sqrt{2}}(\mathbf{e}_1 - \mathbf{e}_3)$ (1 point); [10 minutes]
4. Compute the normal and shear components of the traction vector (0.5 point). [5 minutes]

Exercise 3 Consider a steel with bulk modulus $\kappa = 200$ GPa, shear modulus $G = 50$ GPa and yield strength $\sigma_Y = 0.2$ GPa, subject to the strain $\boldsymbol{\varepsilon} = (\lambda - 1)\mathbf{e}_1 \otimes \mathbf{e}_1 + \left(\frac{1}{\sqrt{\lambda}} - 1\right)\mathbf{e}_2 \otimes \mathbf{e}_2$.

1. At what value of the stretch parameter λ will the steel yield, according to the Von Mises criterion? (2 points); [25 minutes]
2. At what value of the stretch parameter λ will the steel yield, according to the Tresca criterion? Explain the difference (if any) with the results obtained with the Von Mises criterion (1.5 points); [20 minutes]