

EXAM FTV1 – RUG  
JUNE 23, 2017

Maximum # points for each question are indicated

**Question 1**

Consider a tank of volume  $V$ , completely filled, well-stirred and with an inlet and an outlet and a volumetric inflow of  $\phi_v$ . The system is at steady-state. The water in the tank contains component  $A$ , with concentration  $c_A$ . At time  $t=0$ , the inflow of  $A$  stops and the inflow of another component  $B$  with (constant) concentration  $c_{B,in}$  is started.

$$V=10 \text{ dm}^3; \phi_v=1 \text{ dm}^3/\text{s}; c_{A,0}=340 \text{ mM} \text{ and } c_{B,in}=200 \text{ mM}$$

- A) Give the mass balances for component  $A$  and  $B$ . 10
- B) Show that under the condition that the rate of change of  $c_A$  is equal but opposite to the rate of change of  $c_B$  (i.e.,  $dc_A/dt = -dc_B/dt$ ) the sum of  $c_A$  and  $c_B$  remains constant. 10
- C) How long does it take before the concentration of  $A$  equals that of  $B$ ? 10

**Answer:**

A)

$$V \frac{dc_A}{dt} = -\phi_v c_A$$

$$V \frac{dc_B}{dt} = \phi_v c_{B,in} - \phi_v c_B$$

B)

$$\frac{dc_A}{dt} = -\frac{dc_B}{dt}$$

$$\frac{\phi_v}{V} c_A = \frac{\phi_v c_{B,in} - \phi_v c_B}{V}$$

$$c_A + c_B = c_{B,in} = \text{constant}$$

C)

Solving (1) and (2) gives:

$$c_A(t) = c_{A,0} \exp\left(-\frac{\phi_v}{V}t\right)$$

$$c_B(t) = c_{B,in} \left(1 - \exp\left(-\frac{\phi_v}{V}t\right)\right)$$

$$c_A(t) = c_B(t)$$

$$t = \frac{V}{\phi_v} \ln\left(1 + \frac{c_{A,0}}{c_{B,in}}\right) = 10 \ln(2.7) \approx 10 \text{ s}$$

**Question 2**

Consider a rectangular aquarium with a volume of  $3 \text{ m}^3$  containing cold-water fish that require the water temperature to be  $10 \text{ }^\circ\text{C}$ . With a (constant) room temperature ( $T_R$ ) of  $22 \text{ }^\circ\text{C}$ , the system needs a cooling element. The water is well stirred. Consider steady-state conditions.

Water density ( $\rho$ ) and specific heat capacity ( $c_p$ ) are  $10^3 \text{ kg}\cdot\text{m}^{-3}$  and  $4.2 \cdot 10^3 \text{ J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$ , respectively.

**A)** Given a total average heat transfer coefficient ( $U$ ) of  $420 \text{ W}/\text{m}^2\text{K}$  and a total surface area for heat exchange of  $6 \text{ m}^2$ , calculate the power of the cooling element. **5**

**B)** The aquarium is replaced with a larger one with length, width and height, all twice as large. Given the same steady-state conditions but a heat transfer coefficient of just  $300 \text{ W}/\text{m}^2\text{K}$  (because of the thicker glass!), what should now be the power of the cooling element? **10**

**C)** Suppose (for the aquarium in **A**), the cooling element is turned off. How much time does it take for the water to warm up to  $16 \text{ }^\circ\text{C}$ ? **15**

### Question 3

4

To study the time scale of CO<sub>2</sub> diffusion into water, we perform the following experiment. A large dish with water (layer thickness is 10 cm) is placed in a closed cabinet filled with pure CO<sub>2</sub> gas. Initially, the water contains no CO<sub>2</sub>. Under the applied experimental conditions, the diffusion coefficient of CO<sub>2</sub> in water ( $D_c$ ) is  $1.69 \cdot 10^{-9} \text{ m}^2/\text{s}$  and the (maximal) solubility of CO<sub>2</sub> in water ( $c^*$ ) is  $1.73 \text{ kg/m}^3$ .

- A) Calculate the penetration depth of CO<sub>2</sub> and the Fourier number after one hour. 5
- B) Is the penetration theory (still) valid after one hour? Explain! 10
- C) Calculate the average CO<sub>2</sub> concentration in the water after one hour while assuming that the very top layer of the water is saturated with CO<sub>2</sub>. 10
- D) What is the time needed to reach an average of 95% CO<sub>2</sub> saturation in the water? 15

**Answer:**

A)

Penetration depth and Fourier number after one hour:

$$\delta = \sqrt{\pi D_c t} = 6.36 \cdot 10^{-3} \text{ m}$$

$$F_0 = \frac{D_c t}{(2D)^2} = 1.5 \cdot 10^{-4}$$

B) Yes, penetration theory valid as long as  $\sqrt{\pi D_c t} \leq \frac{D}{2}$   
 $6.36 \cdot 10^{-3} \ll 0.05$  True!

C)

$$\langle \phi_m'' \rangle = 2 \sqrt{\frac{D_c}{\pi}} (c_s - c_\infty) = 2 \sqrt{\frac{1.69 \cdot 10^{-9}}{\pi \times 3600}} \cdot 1.73 = 1.34 \cdot 10^{-6} \text{ kg/m}^2 \text{ s}$$

$\approx 0$

$$\text{Total flux is : } \langle \phi_m'' \rangle \times t = 1.34 \cdot 10^{-6} \times 3600 = 4.8 \cdot 10^{-3} \text{ kg/m}^2$$

$$\text{Average concentration is : } 4.8 \cdot 10^{-3} / 0.1 = 4.8 \cdot 10^{-2} \text{ kg/m}^3$$

$$\frac{c_s - 0.95c_s}{c_s - 0} = 0.05$$

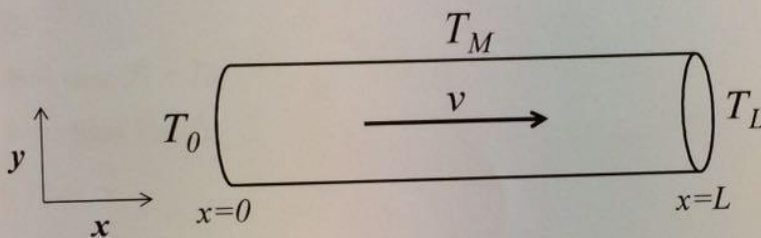
From Fourier plot (average concentration, plane geometry) we obtain a  $F_0$  number of 0.30

$$F_0 = \frac{D_c t}{(2D)^2} = 0.30$$

$t$  is 1972 hrs.

#### Question 4

Consider a water flow of velocity  $v$  through a horizontal, hollow, metal pipe of length  $L$ , cross-sectional area  $A$  and radius  $R$ . At  $x=0$ , the temperature of the water is  $T_0$  and the metal pipe itself has a (constant) temperature of  $T_M$ , with  $T_M > T_0$ . Because of the high  $Re$  number, the water in the pipe is well-stirred, implying  $T$  is a function of  $x$  but not of  $y$ . The overall heat transfer coefficient from the pipe wall to the water is  $h$ .



A) Assuming steady-state, derive an expression for the temperature profile along the length of the pipe. 15

B) What is the temperature at the end of the pipe ( $T_L$ ) if the heat transfer coefficient  $h$  is either close to zero or infinitely high? 5

**Answer:**

A)

$$V\rho c_p \frac{dT}{dt} = \phi_v \rho c_p T|_{in} - \phi_v \rho c_p T|_{out} + \phi_W + \phi_H$$

With  $\phi_v = vA$  and  $dT/dt = 0$

**Answer:**

**A)**

$$V\rho c_p \frac{dT}{dt} = \phi_v \rho c_p T|_{in} - \phi_v \rho c_p T|_{out} + \phi_W + \phi_H$$
$$= 0 \quad = 0 \quad = 0$$

$$\phi_W = -\phi_H = -UA(T - T_R)$$

$$= -420 \times 6 \times (-12) = 30.2 \text{ kW}$$

**B)** The surface area available for heat exchange will be 4 times larger.

$$\phi_W = 300 \times 4 \times 6 \times 12 = 86.4 \text{ kW}$$

**C)**

$$V\rho c_p \frac{dT}{dt} = \phi_H = UA(T_R - T)$$

$$\int_{10}^{16} \frac{dT}{T_R - T} = \frac{UA}{V\rho c_p} \int_0^t dt$$

$$\ln\left(\frac{T_R - 10}{T_R - 16}\right) = \frac{UA}{V\rho c_p} t$$

$$\ln\left(\frac{12}{6}\right) = 0.69 = 210^{-4} t$$

$$t = 3450 \text{ s} \approx 1 \text{ hr}$$

$$0 = v\pi R^2 \rho c_p T \Big|_x - v\pi R^2 \rho c_p T \Big|_{x+dx} + h2\pi R dx (T_M - T)$$

$$-\frac{(T_{x+dx} - T_x)}{dx} = -\frac{2h}{\rho c_p v R} (T_M - T)$$

$$\frac{dT}{dx} = \frac{2h}{\rho c_p v R} (T_M - T)$$

$$\int_{T_0}^T \frac{dT}{T_M - T} = \frac{2h}{\rho c_p v R} \int_0^x dx$$

$$\ln \frac{T_M - T_0}{T_M - T} = \frac{2h}{\rho c_p v R} x$$

$$T(x) = T_M + (T_0 - T_M) \exp\left(-\frac{2h}{\rho c_p v R} x\right)$$

**B)**

$$h=0 \text{ then } T_L = T_0$$

$$h=\infty \text{ then } T_L = T_M$$

**Final mark = total # points/12**