

Re-Examination of Physical Systems

June 24, 2016, 9:00-12:00

INSTRUCTIONS (READ THIS CAREFULLY)

1. **Make sure you solve each problem on a separate sheet. Solutions of different problems appearing on the same sheet may be discarded.**
2. Write your name and student number clearly on top of every sheet.
3. Solutions which are not on the right sheet will not be graded. If you need extra paper, use the sheets provided and indicate for which problem the extra sheet is used for.
4. Solve the problems in a systematic way and check your answers. If you think you have made a mistake in a calculation, indicate this. Argue then how you intended to get the right answer and write this down.
5. This is a closed-book examination. No books, notes or graphical calculators may be used during the examination.
6. **Write in a legible manner. Unreadable text will not be handled during grading.**

Problem #1. Complex Numbers and Differential Equations (10 Points)

Consider the following complex numbers:

$$A = 4 + 2i, B = -1 - i, C = 2 - 5i \text{ and } D = -3 + 4i$$

0) **USE A NEW SHEET!**

- a) Convert A, B, C and D to the Euler notation with $-\pi < \phi < \pi$. **(2 points)**
- b) Calculate $X = A \cdot B \cdot C^*$, $Y = (D^* - C)/B^*$ and $Z = (A \cdot D)/(A^* \cdot D)$ and write them in the Euler notation with $-\pi < \phi < \pi$. **(3 points)**
- c) Draw X, Y and Z in the complex plane. **(1 point)**

Consider the following differential equation:

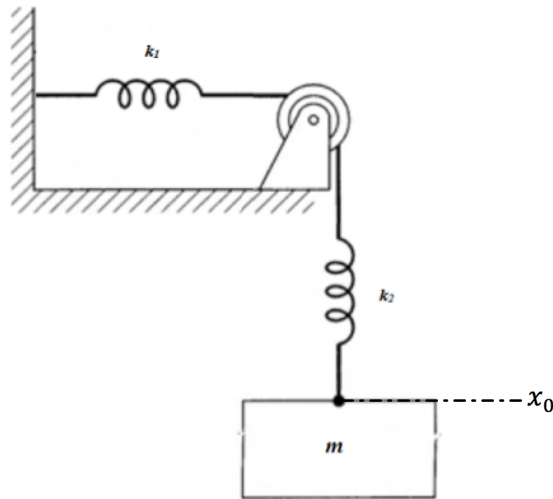
$$y'' + by' + cy = 0$$

and the solution $y = c_1 \exp(-5t) \cos(8t) + c_2 \exp(-5t) \sin(8t)$.

- d) Determine the values of b and c for which the above solution corresponds with the differential equation. **(2 points)**
- e) Solve the initial value problem with $y(0) = 2$ and $y'(0) = -2$ and write down the complete solution for the differential equation. **(2 points)**

Problem #2. Mechanical Oscillations without Damping (17 Points)

A mass m of 100 g is connected to a horizontal and vertical spring, divided by a pulley (see figure 1). At x_0 , the mass is its resting position (equilibrium point) due to gravitational force. The mass is put into a harmonic oscillation with an amplitude of 0.1 m and a maximum acceleration of 5 m/s^2 . The motion is described by: $x(t) = A \sin(\omega t)$. Assume there is no friction.

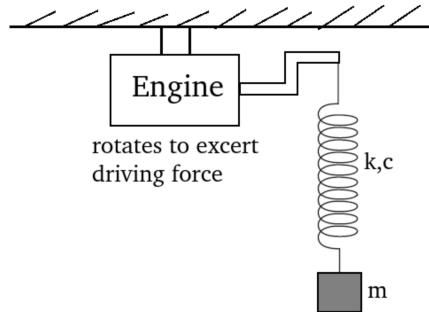


0) **USE A NEW SHEET!**

- a) Why is the gravity not relevant for the oscillation? **(1 point)** Using the *relevant forces*, give the force balance of this system and give the differential equation describing the motion. **(2 points)**
- b) Calculate the spring constants k_1 and k_2 . Assume that $k_1 = 2k_2$. **(5 points)**
- c) Give the positions at which mass m has reached its maximum speed and acceleration. **(2 points)**
- d) Calculate the maximum speed of the mass m . **(3 points)**
- e) The total energy of the system is given by $E_{\text{tot}} = E_{\text{pot}} + E_{\text{kin}} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$. Calculate the total energy of the system. **(4 points)**

Problem #3. Mechanical Oscillations with Damping (17 Points)

A mass m is hanging vertically on a spring. The upper end is connected to a driving engine which is inactive for now. The spring is non-ideal. It has spring constant $k = 50 \text{ N/m}$ and damping constant $c = 10 \text{ Ns/m}$. See the figure below. When the engine is inactive, one can pull the mass downwards after which an oscillation might occur.



- 0) **USE A NEW SHEET!**
- a) Calculate which mass will make the system exactly critically damped. **(3 points)**
- b) If the mass would be a little lighter than the answer you calculated in the previous question, would that produce an underdamped system or an overdamped system? **(3 points)**
- c) Suppose we now turn on the engine to produce a driving force $F = F_0 \cos \omega t$ with $F_0 = 1 \text{ N}$. Calculate the resonance frequency of the system. In case you were unable to answer the first question, use a mass of $m = 0.347 \text{ kg}$. *HINT: the amplitude of the motion is inversely proportional to $\sqrt{(\omega^2 - \omega_0^2)^2 + 4\gamma^2\omega^2}$.* **(3 points)**
- d) Use your answer to the previous question to draw a graph. On the x -axis should be the driving frequency ω and on the y -axis should be the amplitude of the motion of the mass m . **(3 points)**
- e) If we now decide to drive the mass exactly at the natural frequency $\omega = \omega_0$, will there be resonance? Why (not)? **HINT:** In order to answer this question one needs to compare the driving amplitude of the engine with the response amplitude of the mass. The response amplitude is in meters, but the driving amplitude is F_0 , which is in Newtons. Hence one should convert the driving amplitude to meters. This can be done by using $F_0/(m\omega^2)$ for the driving amplitude. **(3 points)**
- f) At $\omega = \omega_0$, will the motion of the mass be a pure harmonic oscillation? Why (not)? **(2 points)**

Problem #4. Electric Circuit – I (18 Points)

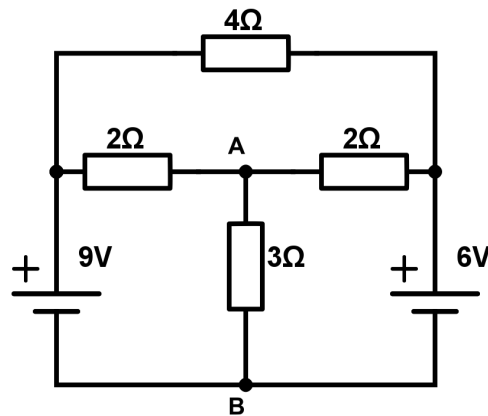
0) **USE A NEW SHEET!**

a) A simple wire-wound resistor is made by winding a metal wire around a non-conductive core. There are two types of wires available with a radius of 0.04 mm:

1. Nichrome (Nickel/Chrome alloy) with $\rho = 1.105 \cdot 10^{-6} \text{ } \Omega\text{m}$ for €0.11/m
2. Constantan (Nickel/Copper alloy) with $\rho = 0.485 \cdot 10^{-6} \text{ } \Omega\text{m}$ for €0.06/m

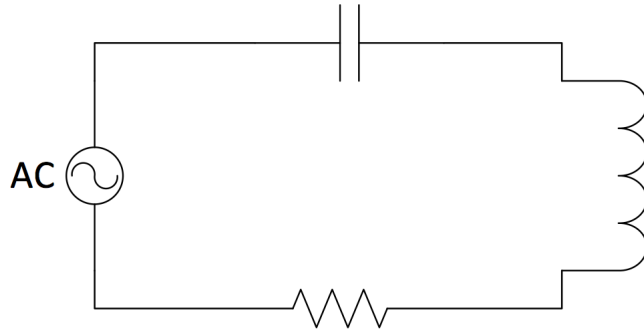
Suppose one would like to make a resistor with a resistance of 100 Ω . What material should be chosen from cost-perspective? **(4 points)**

- b) Give an important physical property that influences the material choice for resistors. **(2 points)**
- c) Give the mathematical representation of both Kirchhoff's laws and explain their physical meaning in one sentence. **(2 points)**



- d) Consider the DC-circuit in the above figure. Write down the KCL and KVL equations that are necessary to obtain the values for the currents through the individual components. Use for instance a figure to show how you defined the currents. **(5 points)**
- e) Calculate the value of the current over the 3 Ω resistor. **(3 points)**
- f) Calculate the voltage difference between points A and B. **(2 points)**

Problem #5. Electric Circuit – II (18 Points)



Consider the above AC-circuit. In this circuit, $R = 10 \Omega$, $L = 500 \text{ mH}$, $C = 50 \text{ nF}$.

- 0) **USE A NEW SHEET!**
- a) What are the complex impedances of the inductor and the capacitor. **(2 points)**
- b) Write down the total complex impedance of the circuit. **(2 points)**
- c) What is the total complex impedance of the circuit when $\omega = \omega_0$, namely when the system is at resonance? **(2 points)**
- d) Calculate $A_c = \frac{V_c}{V_{in}}$ (with real and imaginary parts) and $|A_c| = \left| \frac{V_c}{V_{in}} \right|$. **(3 points)**
- e) Calculate $A_L = \frac{V_L}{V_{in}}$ (with real and imaginary parts) and $|A_L| = \left| \frac{V_L}{V_{in}} \right|$. **(3 points)**
- f) Derive the complete inhomogeneous 2^{nd} order differential equation that describes this system. *Hint: use the Kirchhoff's law for voltage for the entire circuit and make sure you have the current in some form in all terms and proceed with the derivation.* **(2 points)**
- g) What is the value of the resonance frequency, ω_0 ? **(2 points)**
- h) What is the value of the coefficient which is equivalent to γ in a mass-spring system? **(2 points)**

Problem #6. Business Dynamics (20 Points)

0) **USE A NEW SHEET!**

a) Describe/explain how the bullwhip effect can occur by using an example product/supply chain. An example could be (but you may use one of your own) the production and supply chain of single malt whisky.

Hint: single malt whisky, other than blended whisky, is aged for years in specific wooden casks and cannot be mixed with the contents of other casks in case of high demand. (5 points)

b) Imagine a production process of Glenhoddle™ single malt whisky. It consists of 7 steps: Malting, Grinding, Brewing, Fermentation, Distillation, Ageing and Bottling. The production site currently has capacity for 2,400 bottles to age at the same time. The ageing process takes 12 years and is identified as the bottleneck of the production process. Calculate the maximum throughput rate that can be realised in the best case scenario. **(3 points)**

c) Glenhoddle™ sees demand increasing and wishes to increase ageing capacity, to deliver 300 bottles yearly. Give the required percentage increase in capacity. What is a necessary condition for increasing throughput rate? **(3 points)**

d) Give Little's Law (the equation) **(2 points)**. Then explain the equation in no more than 2 sentences. **(1 point)**

e) Oscillations in a supply chain can be described by the following formula:

$$\frac{d^2 q_i(t)}{dt^2} + \frac{\beta + \epsilon}{T} \frac{dq_i(t)}{dt} + \frac{1}{T\tau} q_i(t) = \frac{1}{T} \left(\frac{1}{\tau} q_{i+1}(t) + \beta \frac{dq_{i+1}(t)}{dt} \right) \quad (1)$$

which shows significant similarities with the formula for damped harmonic (mechanical) oscillations:

$$\frac{d^2 x}{dt^2} + 2\gamma \frac{dx}{dt} + \omega_0^2 x = \frac{F_0}{m} \cos(\omega t) \quad (2)$$

Use the solutions (eigenvalues) for the mass-spring-damper mechanical system given by

$$\lambda_{1,2} = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2}$$

to derive the three forms of oscillations in the case of business dynamics **(4.5 points)** and express the boundary condition for a stable supply chain (i.e. damping occurs). **(1.5 points)**