

Solid Mechanics (for IEM)

WBIE055-05

Resit Written Exam (2 hours)

Exercise 1

Provide an example of equibiaxial strain in small deformations.

1. What are the eigenvalues of the associated strain tensor?
(0.5 point) [5 minutes]
2. What are the values of the minimum and of the maximum strain?
(0.5 point) [5 minutes]
3. What is the volume expansion associated with this strain state?
(0.5 point) [5 minutes]

Exercise 2

Consider the hydrostatic stress state with pressure $p = 150$ MPa.

1. Determine the traction vector associated with this stress state, acting on a surface with normal parallel to $\mathbf{d} = \mathbf{e}_1 - 2\mathbf{e}_2$; (1 point) [10 minutes]
2. For the same surface normal, compute the normal (σ_n) and shear stress (τ) components of the traction vector; (0.5 point) [5 minutes]
3. Identify a plane normal for which the shear stress component is not zero. Explain your result. (0.5 point) [5 minutes]

Exercise 3

Consider the displacement field

$$\mathbf{u}(\mathbf{x}) = 10^{-3}(x_1\mathbf{e}_1 + x_2\mathbf{e}_1 + x_3\mathbf{e}_2) . \quad (1)$$

1. Compute the displacement gradient; (1 point) [10 minutes]

2. Compute the stress tensor in the case of isotropic linear elasticity;
(1.5 point) [15 minutes]
3. Indicate the magnitude of the largest stress component, in the basis $\mathbf{e}_i$
($i = 1, 2, 3$), by assuming a shear modulus $\mu = 70$ GPa and Young's
modulus $E = 200$ GPa. (0.5 point) [10 minutes]

Exercise 4

Consider the deformation shown in Figure 1 (strain components are zero out of the (x_1, x_2) plane) of a linear elastic material.

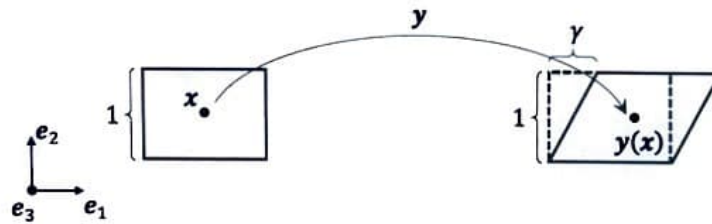


Figure 1: Deformation map $\mathbf{y}(\mathbf{x})$.

1. Determine the matrix representation of the deformation gradient $\mathbf{F}$, in the basis specified in Figure 1, and the relative volume change (1 point) [10 minutes];
2. For this same deformation, compute the matrix representation of the Cauchy stress tensor (1.5 points) [20 minutes];
3. Consider the following yield surface

$$f = \tau - \tau_Y \quad (2)$$

with $\tau_Y = 0.25$ GPa and $\tau = \boldsymbol{\sigma} : \mathbf{s} \otimes \mathbf{n}$. If $\mathbf{s} = \frac{1}{\sqrt{5}}(2\mathbf{e}_1 + \mathbf{e}_2)$ and $\mathbf{n} = \frac{1}{\sqrt{5}}(2\mathbf{e}_2 - \mathbf{e}_1)$, at which value of γ will the material start to deform plastically? (1.5 points) [20 mins].