

1 // Bernoulli between 1 and 4

A) $\dot{\Phi}_v$ of single pipe is $125,6/20 \text{ m}^3/\text{s}$

$$\dot{\Phi}_v = v \cdot A \quad v = 6,28/3,14 = 2 \text{ m/s}$$

$$0 = \dot{\Phi}_m \left\{ \underbrace{\frac{1}{2} v_1^2}_{\approx 0} + \underbrace{\frac{p_1}{\rho}}_{=0} + gh_1 - \underbrace{\frac{1}{2} v_4^2}_{=0} - \underbrace{\frac{p_4}{\rho}}_{=0} - gh_4 \right\} + \dot{\Phi}_W$$

$$0 = 6,28 \cdot 10^3 \left(10 \times 10 - \frac{1}{2} 2^2 \right) + \dot{\Phi}_W$$

$$\dot{\Phi}_W = -0,615 \text{ MW}$$

$$\text{Total energy} = 20 \times -0,615 = -12,3 \text{ MW}$$

< 0 because energy is extracted

B) Smooth pipes \rightarrow Blasius (or Moody-plot)

$$4f = 0,316 \times Re^{-1/4}$$

$$Re = \frac{\rho \cdot v \cdot D}{\eta} = \frac{10^3 \times 2 \times 2}{10^{-3}} = 4 \cdot 10^6$$

$$D = \left(\frac{4A}{\pi} \right)^{1/2} = 2 \text{ m}$$

$$4f = 0,0224 \times 0,316 = 0,0071$$

$$e_{FR} = 4f \cdot \frac{1}{2} \langle v^2 \rangle \cdot \frac{L}{D}$$

$$= 0.0071 \times 2 \times \frac{50}{2} = 0,355$$

$$e_{FR} \cdot \phi_m = 0,355 \times 6,28 \times 10^3 = 2,23 \text{ kW}$$

Energy generated by a single turbine is 615 kW

Losses are just 0,36%!

$$F_D = K \cdot v^a \cdot D^b \cdot \rho^c \cdot \eta^d$$

2) 1)

$$\frac{\text{kg} \cdot \text{m}}{\text{s}^2} = \left(\frac{\text{m}}{\text{s}}\right)^a \text{m}^b \left(\frac{\text{kg}}{\text{m}^3}\right)^c \left(\frac{\text{kg}}{\text{m} \cdot \text{s}}\right)^d$$

$$\text{kg: } 1 = c + d \quad a = 2 - d$$

$$\text{m: } 1 = a + b - 3c - d \quad b = 2 - d$$

$$\text{s: } -2 = -a - d \quad c = 1 - d$$

$$F_D = K \cdot v^{2-d} \cdot D^{2-d} \cdot \rho^{1-d} \cdot \eta^d$$

$$\frac{F_D}{v^2 \cdot D^2 \cdot \rho} = K \left(\frac{\eta}{v \cdot D \cdot \rho}\right)^d (= K \cdot \text{Re})$$

Check with π -theorem:

$$\# \text{ of dimensionless groups} = 5 - 3 = \underline{\underline{2}}$$

$$B) \frac{\eta_1}{v_1 D_1 \rho_1} = \frac{\eta_2}{v_2 D_2 \rho_2} \quad \eta_1 = \eta_2 \text{ AND } \rho_1 = \rho_2$$

$$v_1 D_1 = v_2 D_2$$

$$v_2 = 10 v_1$$

$$D_2 = \frac{1}{10} D_1$$

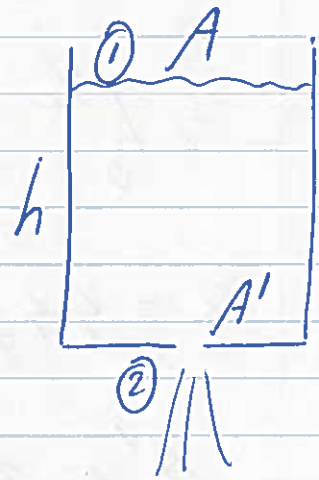
Scaling Factor is $1/10$

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$$A) \frac{dM}{dt} = -\rho_m$$

~~$$\rho \frac{dV}{dt} = -\rho \phi_v = -\rho \cdot v \cdot A'$$~~



$$V = A \cdot h$$

$$A \cdot \frac{dh}{dt} = -v \cdot A'$$

Bernoulli between 1 and 2 gives v

$$\underbrace{\frac{1}{2} v_1^2}_{\approx 0} + gh_1 + \underbrace{\frac{p_1}{\rho}}_{=0} = \frac{1}{2} v_2^2 + gh_2 + \underbrace{\frac{p_2}{\rho}}_{=0}$$

$$gh_1 = \frac{1}{2} v_2^2 \rightarrow v = (2gh)^{1/2}$$

$$A \frac{dh}{dt} = -(2gh)^{1/2} \cdot A'$$

$$\int_{h_0}^0 \frac{dh}{h^{1/2}} = -(2g)^{1/2} \cdot \frac{A'}{A} \int_0^{20} dt$$

$$h_0 = \frac{4500 \cdot 10^{-3}}{4.5} = 1 \text{ m}$$

$$-2h_0^{1/2} = -(2g)^{1/2} \cdot \frac{A'}{A} \cdot 20$$

$$2 = 4.47 \times \frac{A'}{4.5} \times 20$$

$$A' \approx 2/20 = ~~10 \text{ cm}^2~~ 0.1 \text{ m}^2$$

Indeed $A' \ll A (=4.5 \text{ m}^2)$

B) v remains the same but ϕ_v
doubles

So time needed is $\frac{20}{3} \approx 7 \text{ sec.}$

$$4) \equiv A) \quad \underbrace{V \frac{dX}{dt}}_{=0} = \cancel{\phi_v} - \cancel{\phi_v} + \phi_w - \phi_h$$

$$\phi_w = \phi_h = U \cdot A \cdot \Delta T = U \cdot A \cdot (T_R - T)$$

$$= 420 \times 6 \times 12 = 30,2 \text{ kW}$$

B) A will be 4x larger

$$\phi_w = 300 \times 4 \times 6 \times 12 = 86,4 \text{ kW}$$

$$C) \quad V \cdot \frac{dX}{dt} = \phi_h \quad X = \rho \cdot c_p \cdot T$$

$$V \cdot \rho \cdot c_p \frac{dT}{dt} = U \cdot A \cdot (T_R - T)$$

$$\int_{10}^{16} \frac{dT}{T_R - T} = \frac{U \cdot A}{V \cdot \rho \cdot c_p} \int_0^t dt$$

$$\ln \frac{T_R - 10}{T_R - 16} = \frac{U \cdot A}{V \cdot \rho \cdot c_p} \cdot t$$

$$t = \frac{420 \times 6}{3 \times 10 \times 4,2 \cdot 10^3} = 2 \cdot 10^{-4}$$

$$\ln \frac{12}{6} = 2 \cdot 10^{-4} \cdot t$$

$$t = 3466 \text{ sec} \approx 1 \text{ hr}$$

$$5) \underbrace{V \frac{dX}{dt}}_{=0} = \phi_v X|_{in} - \phi_v X|_{out} + \cancel{\phi_w} + \phi_h$$

$$X = \rho \cdot c_p \cdot T \quad \phi_v = v \cdot A$$

$$0 = \rho \cdot c_p \cdot v \cdot A \cdot T \Big|_x - \rho \cdot c_p \cdot v \cdot A \cdot T \Big|_{x+dx} + h \cdot 2\pi R \cdot dx \Delta T$$

$$0 = \rho \cdot c_p \cdot v \cdot \pi R^2 (T_x - T_{x+dx}) = -h \cdot 2\pi R \cdot dx (T_M - T)$$

$$\frac{dT}{dx} = \frac{2h}{\rho \cdot c_p \cdot v \cdot R} (T_M - T)$$

$$\int_{T_0}^T \frac{dT}{T_M - T} = \frac{2h}{\rho \cdot c_p \cdot v \cdot R} \int_0^x dx$$

$$\ln \frac{T_M - T_0}{T_M - T} = \frac{2h}{\rho \cdot c_p \cdot v \cdot R} \cdot x$$

$$\frac{T_M - T}{T_M - T_0} = \exp\left(-\frac{2h}{\rho \cdot c_p \cdot v \cdot R} x\right)$$

$$T(x) = T_M + (T_0 - T_M) \exp\left(-\frac{2h}{\rho \cdot c_p \cdot v \cdot R} x\right)$$

Check $x=0 \quad T(x) = T_0$

$x=\infty \quad T(x) = T_M$

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